Practical cube-attack against **nonce-misused** Ascon

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In this talk

Ascon rationale, its internal components and our attack setting

Cube attack, main problems, first part of the answer

Conditional cubes, second part of the answer

Overview of the **internal-state recovery**

Authenticated encryption

→ one of the winners of CAESAR (2014 – 2019).

Lightweight

> *"meets the needs of most use cases where lightweight cryptography is required"* [*NIST webpage*]

→ winner of NIST LWC standardization process (2018 – 2023).

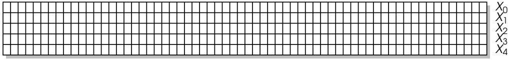
Permutation-based

Duplex Sponge mode [BDPA11] instantiated with permutation $p\colon \mathbb{F}_2^{320} \to \mathbb{F}_2^{320}$.

The permutation

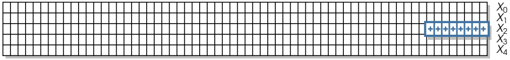
A confusion/diffusion structure…

State

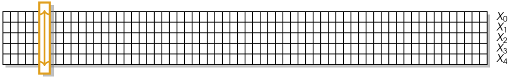


$$p = p_L \circ p_S \circ p_C$$

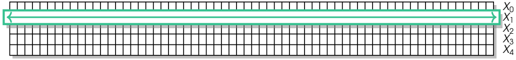
Constant addition p_C



Substitution layer p_S



Linear layer p_L



…studied algebraically

$$\begin{aligned}
y_0 &= \mathbf{x_4 x_1} + x_3 + \mathbf{x_2 x_1} + x_2 + \mathbf{x_1 x_0} + x_1 + x_0 \\
y_1 &= x_4 + \mathbf{x_3 x_2} + \mathbf{x_3 x_1} + x_3 + \mathbf{x_2 x_1} + x_2 + x_1 + x_0 \\
y_2 &= \mathbf{x_4 x_3} + x_4 + x_2 + x_1 + 1 \\
y_3 &= \mathbf{x_4 x_0} + x_4 + \mathbf{x_3 x_0} + x_3 + x_2 + x_1 + x_0 \\
y_4 &= \mathbf{x_4 x_1} + x_4 + x_3 + \mathbf{x_1 x_0} + x_1
\end{aligned}$$

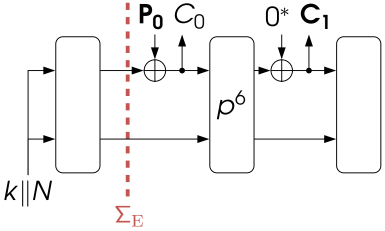
Algebraic Normal Form (ANF) of the S-box

$$\begin{aligned}
X_0 &= X_0 \oplus (X_0 \ggg 19) \oplus (X_0 \ggg 28) \\
X_1 &= X_1 \oplus (X_1 \ggg 61) \oplus (X_1 \ggg 39) \\
X_2 &= X_2 \oplus (X_2 \ggg 1) \oplus (X_2 \ggg 6) \\
X_3 &= X_3 \oplus (X_3 \ggg 10) \oplus (X_3 \ggg 17) \\
X_4 &= X_4 \oplus (X_4 \ggg 7) \oplus (X_4 \ggg 41)
\end{aligned}$$

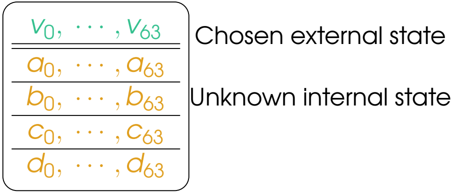
ANF of the linear layer p_L

The nonce-misuse scenario

Simplified setting of Ascon -128



Initialization Encryption



Σ_E State before encryption

- Many reuse of the same (k, N) pair.
- State recovery = compromised confidentiality without interaction.
- No trivial key-recovery nor forgery in that case.
- Different from the generic attack [VV18].

If $v = (v_1, \cdots, v_n)$ and $u = (u_1, \cdots, u_n)$ we define $v^u := \prod_{i=1}^n {v_i}^{u_i}$.

Coefficients $\leftrightarrow$ values relations

Let $f : \mathbb{F}_2^n \to \mathbb{F}_2, v \mapsto \sum_{u \in \mathbb{F}_2^n} \alpha_u v^u$. $\forall\ y \in \mathbb{F}_2^n$ $f(y) = \sum_{u \preceq y} \alpha_u$ and

$$\boxed{\alpha_y = \sum_{u \preceq y} f(u)}$$

Proof.

$$v^u = 1 \iff \mathrm{Supp}(u) \subset \mathrm{Supp}(v)$$

$$\sum_{u \preceq y} f(u) = \sum_{u \preceq y} \sum_{v \preceq u} \alpha_v = \sum_{v \preceq y} \sum_{v \preceq u \preceq y} \alpha_v = \sum_{v \preceq y} 2^{\mathrm{w}(y) - \mathrm{w}(v)} \alpha_v = \alpha_y$$

$\square$

$\implies$ Recovery of α_u for $2^{w(u)}$ chosen queries.

Cube attack principle

f_j: j-th output coordinate, $f_j \in \mathbb{F}_2[a_0, \cdots, a_{63}][v_0, \cdots, v_{63}]$.

$$f_j = \sum_{(u_0,\cdots,u_{63})\in\mathbb{F}_2^{64}} \alpha_{u,\,j} \left(\prod_{i=0}^{63} {v_i}^{u_i}\right), \text{ where } \alpha_{u,\,j} \in \mathbb{F}_2[a_0, \cdots, a_{63}].$$

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Cube attack

Polynomial expression of $\alpha_{u,\,j}$ + value of $\alpha_{u,\,j}$

$=$

equation in unknown variables

$\simeq$

recovery of some information

- Online recovery of the value: $\alpha_{u,\,j} = \sum_{v \preccurlyeq u} f_j(v)$ for $2^{w(u)}$ chosen queries.
- Offline recovery of the expression.

Main problems with the polynomials recovery

Problem 0: impossible access to the full ANF.

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Too many combinatorial possibilities.

$$v_0 v_1 = v_0 \times v_1 = (v_0 v_1) \times 1 = (v_0 v_1) \times v_0 = (v_0 v_1) \times v_1 = (v_0 v_1) \times (v_0 v_1)$$

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We want to solve the system! (Linear, sparse, low-degree, ...)

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Problem 2: finding $\alpha_{u,\,j}$ with “simple” expressions.

We want to solve the system! (Linear, sparse, low-degree, ...)

- Highest-degree terms (degree 2^{t-1} at round t) are easier to study!

Strong constraint: products of two highest-degree terms one round before.

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A partial answer: highest-degree terms

Strong constraint: products of two former highest-degree terms.

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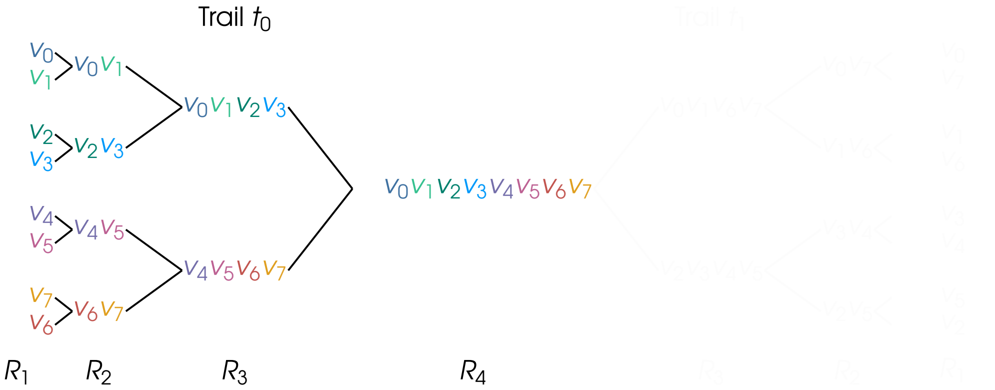

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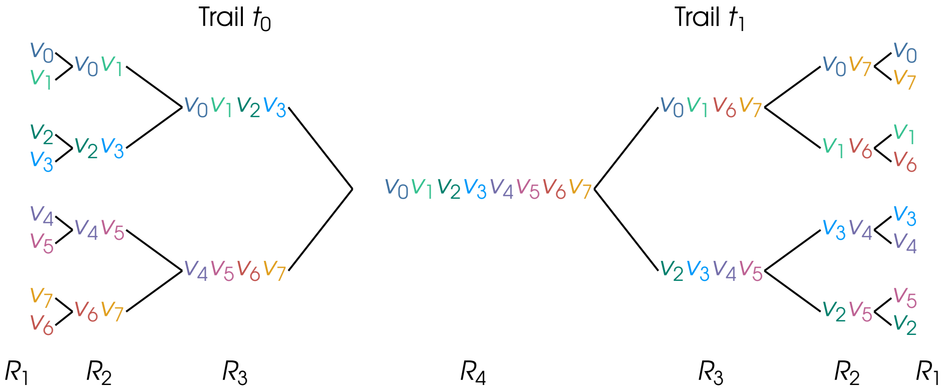

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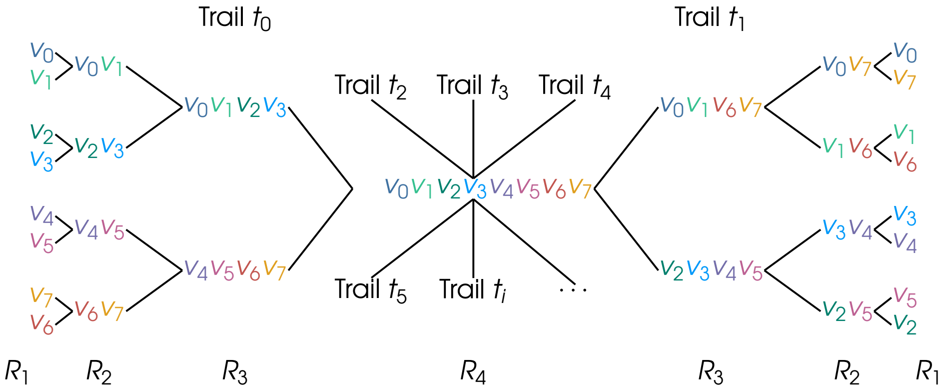

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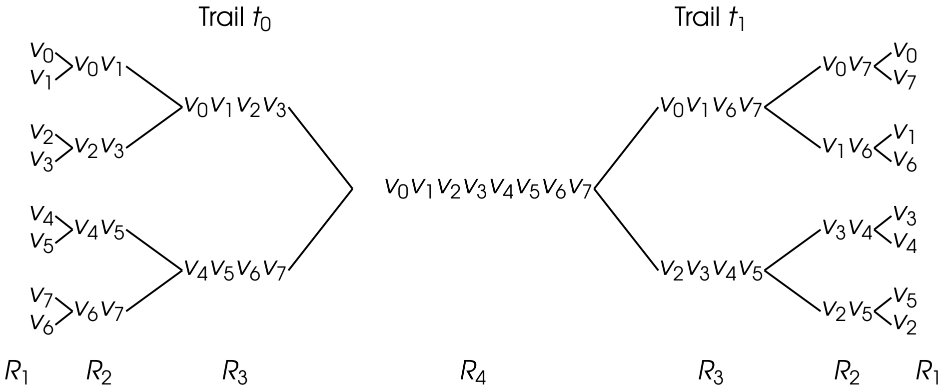

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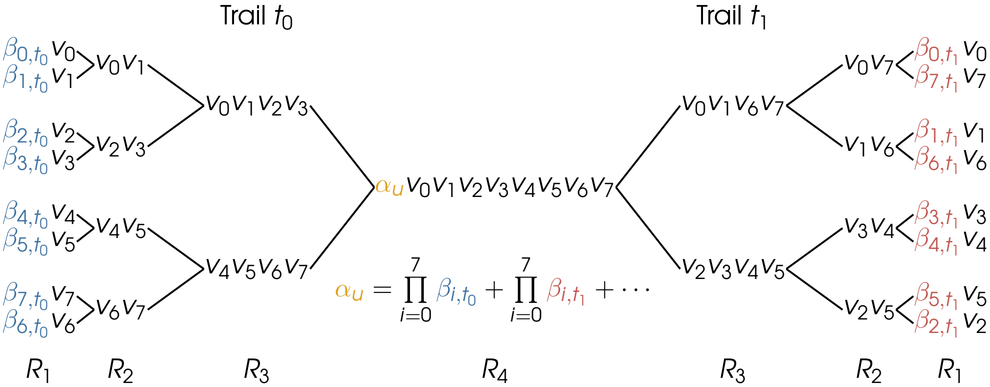
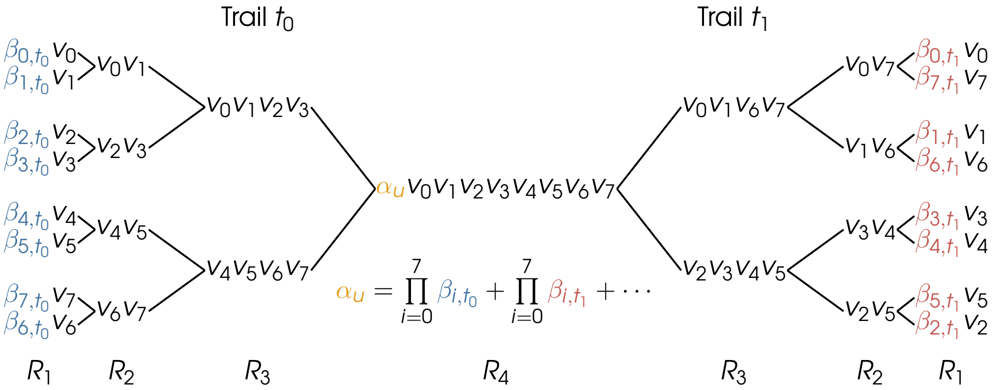


$$\alpha_u = \prod_{i=0}^{7} \beta_{i,t_0} + \prod_{i=0}^{7} \beta_{i,t_1} + \cdots$$

Strong constraint: products of two former highest-degree terms.



For r = 6, still too many trails and α_u usually looks horrible!

- Cheaper / easier recovery: conditional cubes [HWX+17, LDW17, CHK22]

Conditional cube

Look for $\alpha_u = \beta_0 P$ where β_0 simple and known, P unknown.

- Partial knowledge but still: $\alpha_u = 1 \implies \beta_0 = 1$.
- If β_0 is linear, we get a linear system.

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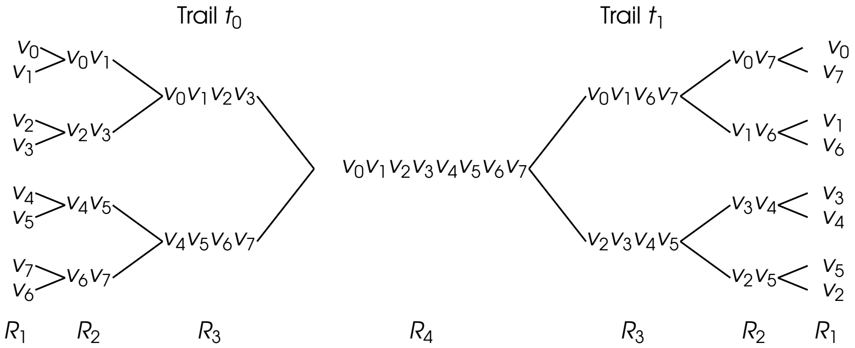

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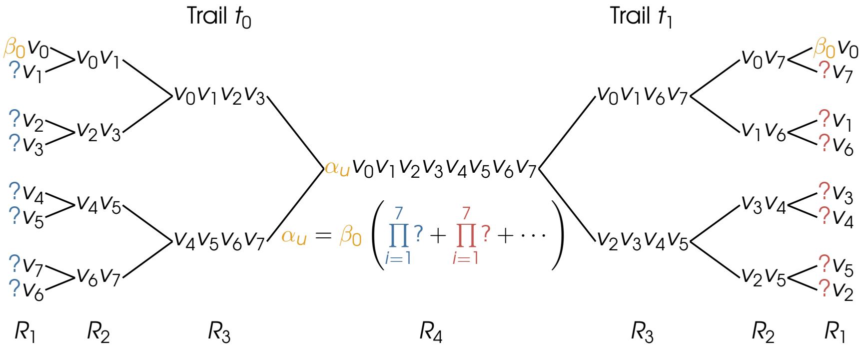

1st round

$$\begin{array}{|c|}\hline v_0 \\ \hline a_0 \\ b_0 \\ c_0 \\ d_0 \\ \hline\end{array} \xrightarrow[S_1]{} \begin{array}{|c|}\hline (a_0+1)v_0 + \cdots \\ \hline v_0 + \cdots \\ \hline \cdots \\ \hline (c_0+d_0+1)v_0 + \cdots \\ \hline a_0 v_0 + \cdots \\ \hline\end{array} \begin{array}{l} \longleftarrow \beta_0 := a_0 + 1 \\ \\ \\ \longleftarrow \gamma_0 := c_0 + d_0 + 1 \\ \\ \end{array}$$

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\begin{array}{l} \longleftarrow \beta_0 := a_0 + 1 \\ \\ \\ \longleftarrow \gamma_0 := c_0 + d_0 + 1 \\ \\ \end{array}
$$

2nd round

A priori: $\forall i \neq 0 \ \big(\beta_0 P + 1Q + \gamma_0 R + (\beta_0 + 1)S\big) v_0 v_i$.

But for some i: $\beta_0 P$ or $\gamma_0 R$!
(Diffusion has just started)

Choosing conditional cubes by forcing linear divisors

1st round

$$\begin{array}{|c|}\hline v_0 \\ \hline a_0 \\ b_0 \\ c_0 \\ d_0 \\ \hline\end{array} \xrightarrow{S_1} \begin{array}{|c|}\hline (a_0+1)v_0+\cdots \\ \hline v_0+\cdots \\ \hline \cdots \\ \hline (c_0+d_0+1)v_0+\cdots \\ \hline a_0v_0+\cdots \\ \hline\end{array} \begin{array}{l} \longleftarrow \beta_0 := a_0+1 \\ \\ \longleftarrow \gamma_0 := c_0+d_0+1 \\ \\ \end{array}$$

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6th round

- With chosen u, $\alpha_{u,\,j} = \beta_0(\ldots) + \gamma_0(\ldots)$, for all output coordinates.

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6th round

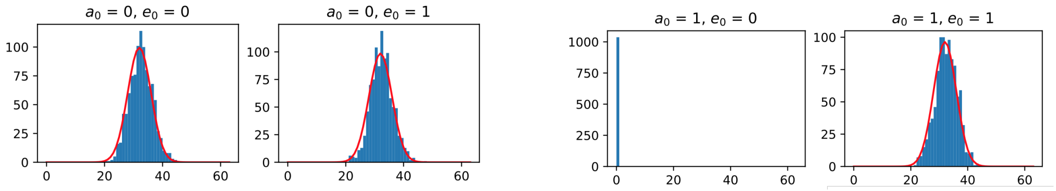
- With chosen u, $\alpha_{u,j} = \beta_0(\dots) + \gamma_0(\dots)$, for all output coordinates.
- $(\alpha_{u,0}, \cdots, \alpha_{u,63}) \neq (0, \cdots, 0) \implies \beta_0 = 1$ or $\gamma_0 = 1$

6th round

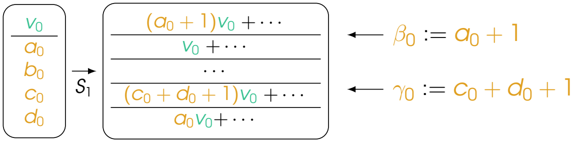
- With chosen u, $\alpha_{u,j} = \beta_0(\dots) + \gamma_0(\dots)$, for all output coordinates.
- $(\alpha_{u,0}, \cdots, \alpha_{u,63}) \neq (0, \cdots, 0) \implies \beta_0 = 1$ or $\gamma_0 = 1$

In practice, reciprocal also true!

$[\alpha_{u,j} = 0, \ \forall j] \implies \beta_0 = 0$ and $\gamma_0 = 0$



Step 1, non-adaptative: 32-degree conditional cubes



Output: e_i for all $i \in \{0,\ldots,63\}$ and a_i for some $i \in \{0,\ldots,63\}$

for all $i \in \{0,\ldots,63\}$ **do**
  $a_i \leftarrow -1, e_i \leftarrow -1$ ▷ Initialize all variables.
end for

for all $i \in \{0,\ldots,63\}$ **do**
  $Z_v \leftarrow \texttt{CubeSumVector}(x^{v \ggg i})$
  if $Z_v = (0,\cdots,0)$ **then**
    $a_i \leftarrow 1, c_i + d_i \leftarrow 1$ ▷ Assumption 1
  else
    $Z_w \leftarrow \texttt{CubeSumVector}(x^{w \ggg i})$
    if $Z_w = (0,\cdots,0)$ **then**
      $a_i \leftarrow 0, c_i + d_i \leftarrow 1$ ▷ Assumption 2
    else
      $c_i + d_i \leftarrow 0$ ▷ No assumption
    end if
  end if
end for

$\Longrightarrow$ Recovery of all $c_i + d_i$, and half of the a_i for $2 \times 64 \times 2^{32} = 2^{39}$

Steps 2 and 3

Step 2, adaptative: 32-degree cubes

$$
\begin{array}{c}
v_0 \\ \hline a_0 \\ b_0 \\ c_0 \\ d_0
\end{array}
\xrightarrow[S_1]{}
\begin{array}{c}
(a_0+1)v_0 + \cdots \\ \hline
v_0 + \cdots \\ \hline
\cdots \\ \hline
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\end{array}
\quad
\begin{array}{l}
\longleftarrow \beta_0 := a_0 + 1 \\
\\
\\
\longleftarrow \gamma_0 := c_0 + d_0 + 1 \\
\\
\end{array}
$$

- The coefficients of 32-degree terms depend only on a_i and $c_i + d_i$.
- Step 1 $\implies$ coefficients α_u drastically simplifies.
- Simple-enough to be effectively-solved (Cryptominisat, [SNC09]).

▶ Recovery of the remaining a_i.

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Step 3, adaptative: **31-degree cubes**

- The remaining unknowns are hidden in the constant terms after 1 round.
- Same principle as Step 2, but with quadratic equations in b_i, c_i.
- ▶ Recovery of all b_i and c_i.

Conclusion

- Full-state recovery on the full 6-round encryption.
- About 2^{40} online time and data, but nonce-misuse.
- Hard to study the complexity of the solving of equations. However effective.
- Does not threaten Ascon directly ... if used properly!

Main questions/openings

- Be careful with implementation : nonce $\neq$ constant!
- Can it lead to key-recovery or forgery attacks?
- Free counter-measure : changing the external state row.

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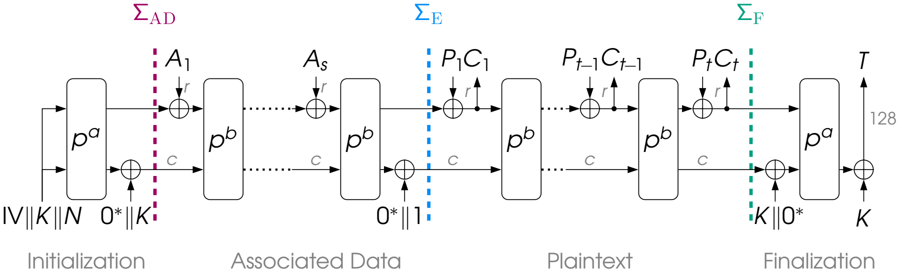
Thank you for
your attention!

The whole Ascon AEAD mode



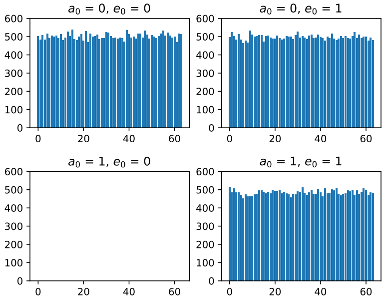
[DEMS, Jea16]

Justifying the "in practice" reciprocal

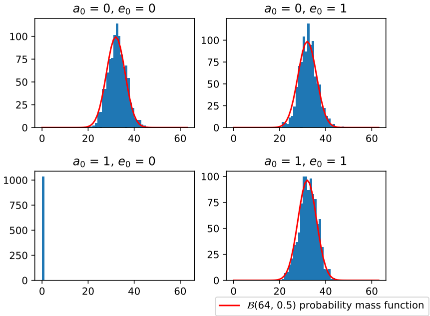
$$\alpha_{u,\,j} = (a_0 + 1)p_{j,1} + (c_0 + d_0 + 1)p_{j,2}\ \forall j \in [\![0, \cdots, 63]\!].$$

When $(a_0 + 1, c_0 + d_0 + 1) \neq (0, 0)$, $\alpha_{u,\,j}$ are not expected to be **all** canceled at the same time.

Whenever we observe that $\alpha_{u,\,j} = 0\ \forall j$, we guess that $(a_0, c_0 + d_0) = (1, 1)$.



Individual cancellations of each $\alpha_{u,j}$
(1000 random internal states)



Hamming weight of the cube-sum vectors
(1000 random internal states)

Counter-Measure: Changing the Input Row

State after initialization	Linear terms after S_1	Size of the sets	Analysis
a_0	$(a_0 + b_0 + d_0 + 1)v_0$	5	
v_0	$(b_0 + c_0 + 1)v_0$	3	
b_0	v_0		$5 + 3 + 5 + 12 < 31$
c_0	v_0		No conditional cube
d_0	$(a_0 + d_0 + 1)v_0$	5	as we describe.
Nb of variables not multiplied by v_0 after S_2		12	
a_0	$(b_0 + 1)v_0$	4	
b_0	$(b_0 + c_0 + 1)v_0$	6	$4 + 6 + 23 > 31$.
v_0	v_0		Cubes can be built as
c_0	v_0		described but less effective.
d_0	*		
Nb of variables not multiplied by v_0 after S_2		23	(32 of the 256-bit state in avg.)

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a_0	v_0		
b_0	$(b_0 + c_0 + 1)v_0$	3	
c_0	$d_0 v_0$	4	$3 + 4 + 5 + 12 < 31$
v_0	$(a_0 + 1)v_0$	5	No conditional cube
d_0	v_0		as we describe.
Nb of variables not multiplied by v_0 after S_2		12	
a_0	$b_0 v_0$	5	
b_0	v_0		$5 + 4 + 5 + 5 + 12 = 31$
c_0	$(d_0 + 1)v_0$	4	but b_0 and $b_0 + 1$ cannot
d_0	$(a_0 + 1)v_0$	5	be used at the same time.
v_0	$(b_0 + 1)v_0$	5	
Nb of variables not multiplied by v_0 after S_2		12	No conditional cube as we describe.

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